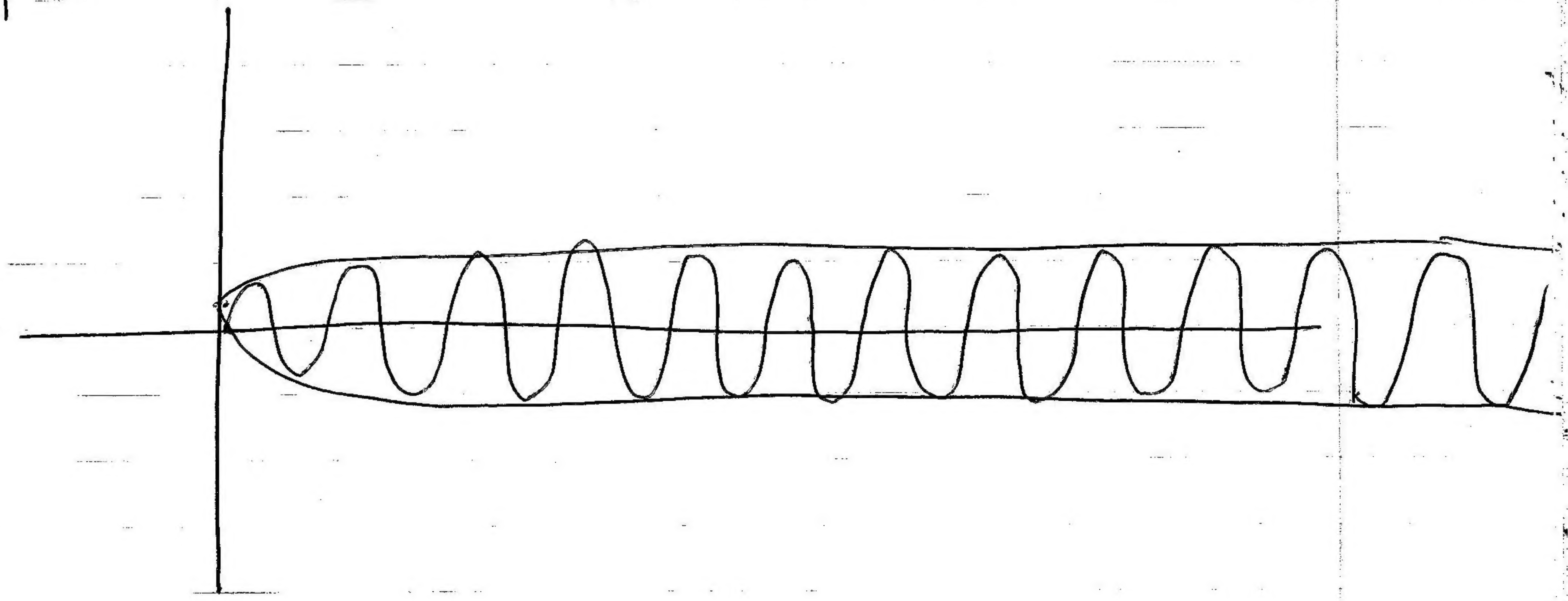
Frequency Response Methods

The steady state Response of a system to a sinusoidal input



$\boxed{1.}$ Bode diagram $\boxed{2.}$ Polar-plot

$\boxed{1.}$ Bode Diagram

* Steps to plot Bode diagram

1. Obtain the total transfer function $G(s)$
2. Find $G(j\omega) \Rightarrow G(j\omega) = Re + Im\,j$
3. obtain $\boxed{|G(j\omega)| = \sqrt{Re^2 + Im^2}}$ & $\boxed{\phi = \tan^{-1}\left(\frac{Im}{Re}\right)}$
4. Obtain the logarithemic gain

$= 20 \log |G(j\omega)|$ in dB

$\boxed{dB = \text{deci Bell}}$

$\log \frac{P_2}{P_1} = \boxed{---}$ [Bell]

$\log \frac{V_2^2}{V_1^2} = \boxed{---}$ [Bell]

$\log \frac{V_2^2}{V_1^2} = 2 \log \frac{V_2}{V_1} = 20 \log \frac{V_2}{V_1} = \boxed{--}$ [dB]

Ex Plot the Bode diagram of the following systems :-

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1. $G(s) = 4$

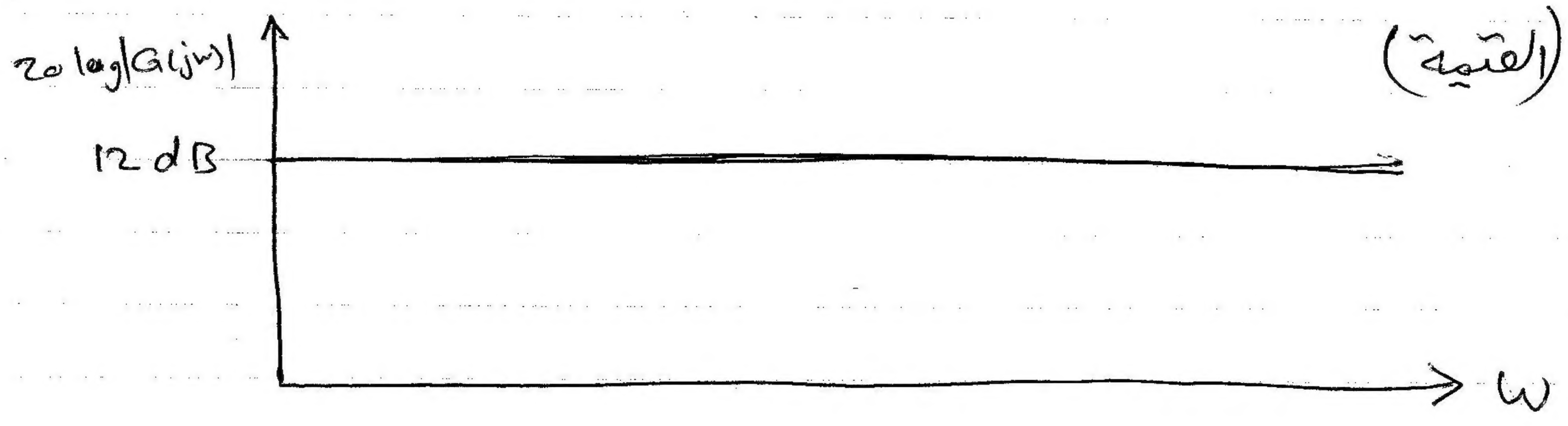
① $G(s) = 4$

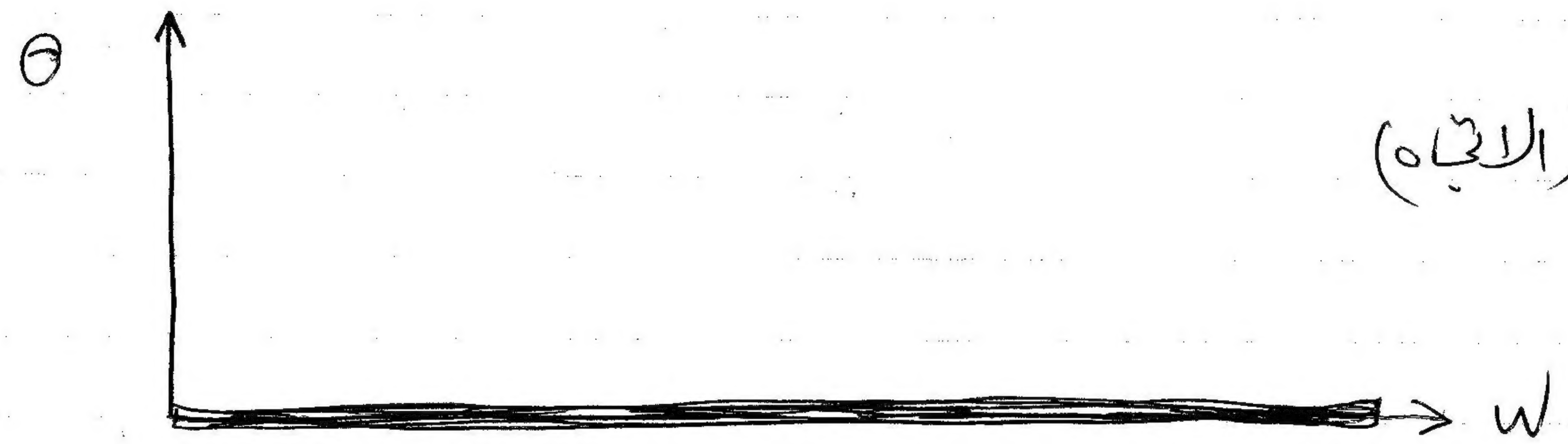
② $G(jw) = 4$

③ $|G(jw)| = 4$

$\theta = \tan^{-1} \frac{Im}{Re} = \tan^{-1} \frac{0}{4} = \tan^{-1} 0 = \boxed{Zero}$, 180, ---

④ $20 \log |G(jw)| = 20 \log 4 = 12\ dB$





$\boxed{2.}$ $G(s) = -4$

① $G(s) = -4$

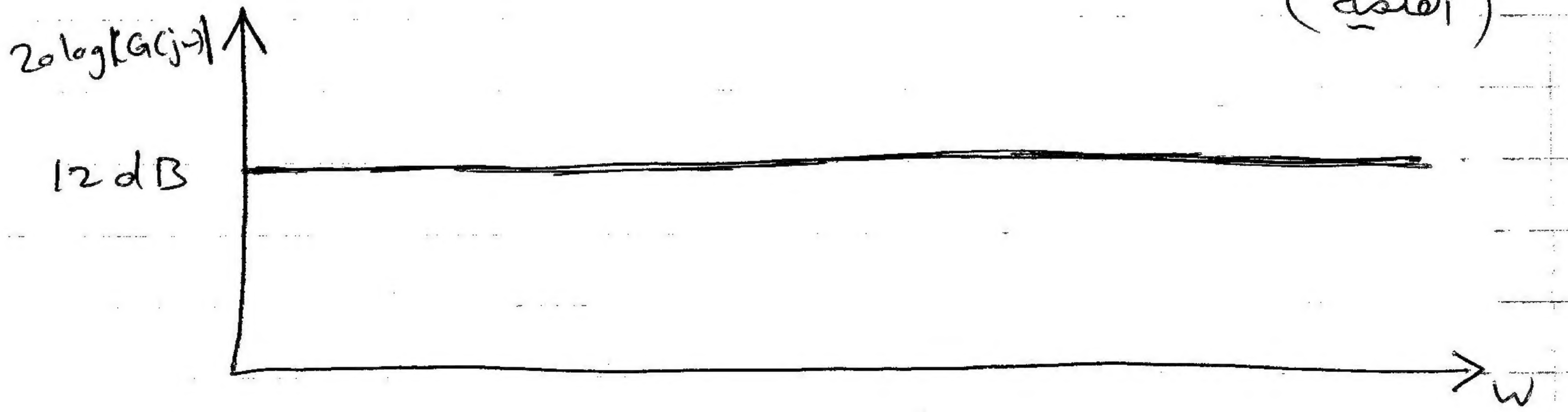
② $G(j\omega) = -4$

③ $|G(j\omega)| = 4$

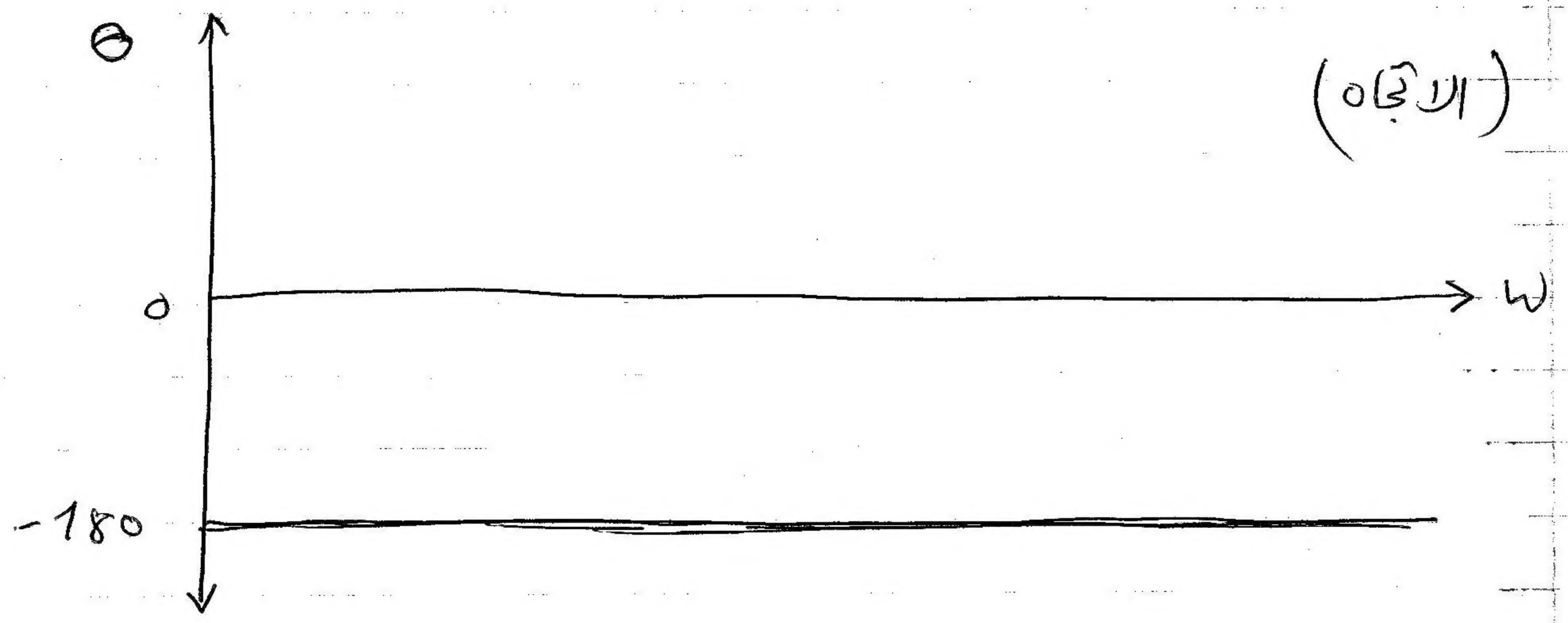
$\theta = \tan^{-1}\frac{Im}{Re} = \tan^{-1}\frac{0}{-4} = \tan^{-1} 0 = \boxed{180^{\circ}}$

-0.1	0	0.1
-5.7	0 180	5.7

(المقدار)



(الاتجاه)



$\boxed{3.}$ $G(s) = S$

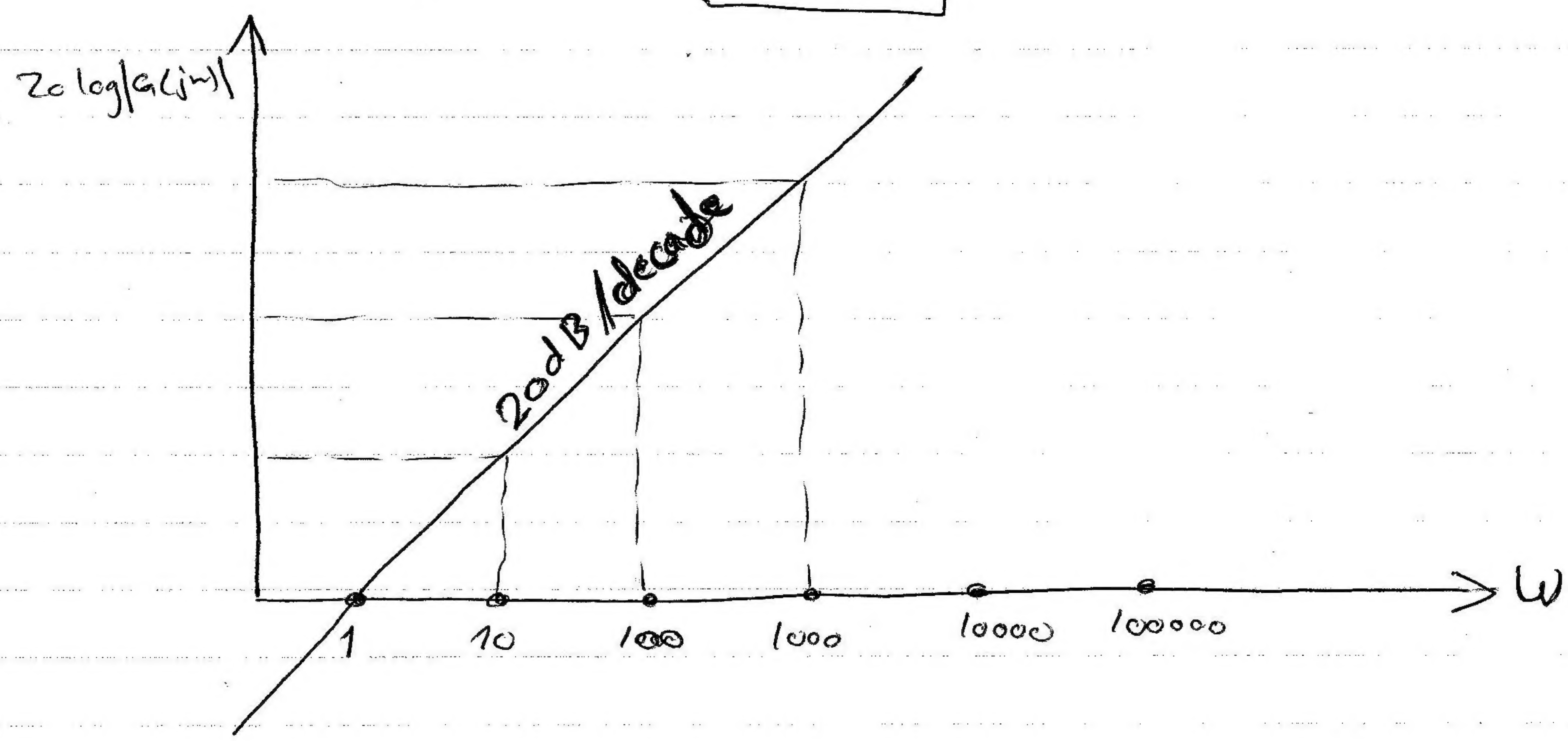
① $G(s) = S$

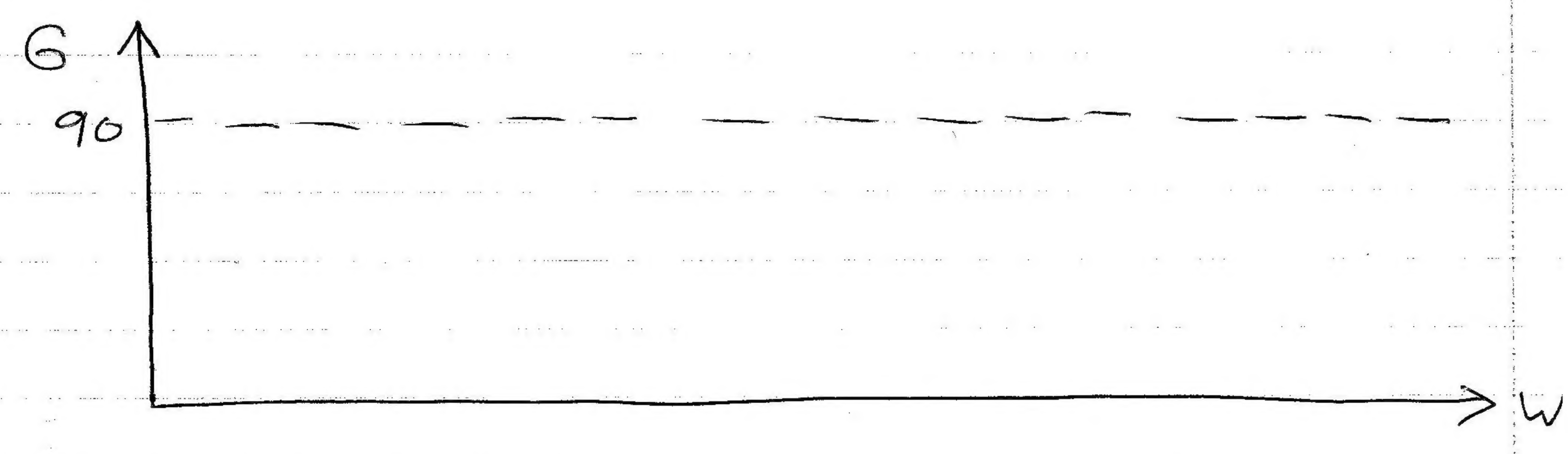
② $G(jw) = jw$

③ $|G(jw)| = w$

$\phi = \tan^{-1} \frac{Im}{Re} = \tan^{-1} \frac{w}{0} = \tan^{-1} \infty = \boxed{90^\circ}$

④ $20 \log |G(jw)| = \boxed{20 \log w}$





$\boxed{4.}$ $G(s) = S^2$

① $G(s) = S^2$

② $G(j\omega) = (j\omega)(j\omega)^2 = -1\,(\omega^2) = \boxed{-\omega^2}$

③ $|G(j\omega)| = \omega^2$

$\theta = \tan^{-1}\frac{Im}{Re} = \tan^{-1}\frac{0}{\omega^2} = \boxed{180^\circ}$

④ $20\log|G(j\omega)| = 20\log\omega^2 = \boxed{40\log\omega}$

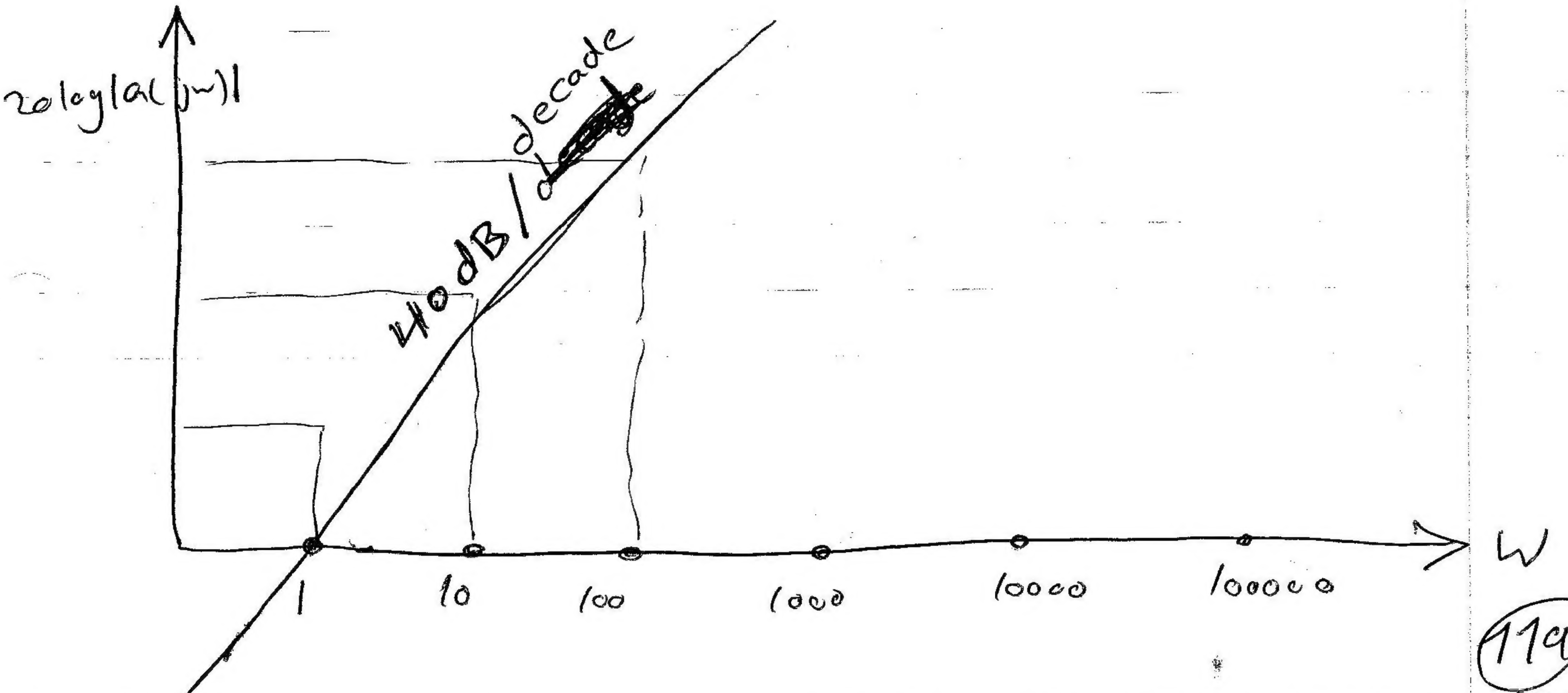
ملاحظة :- كل (S) في البسط يعطينا
phase shift مقدار 90°

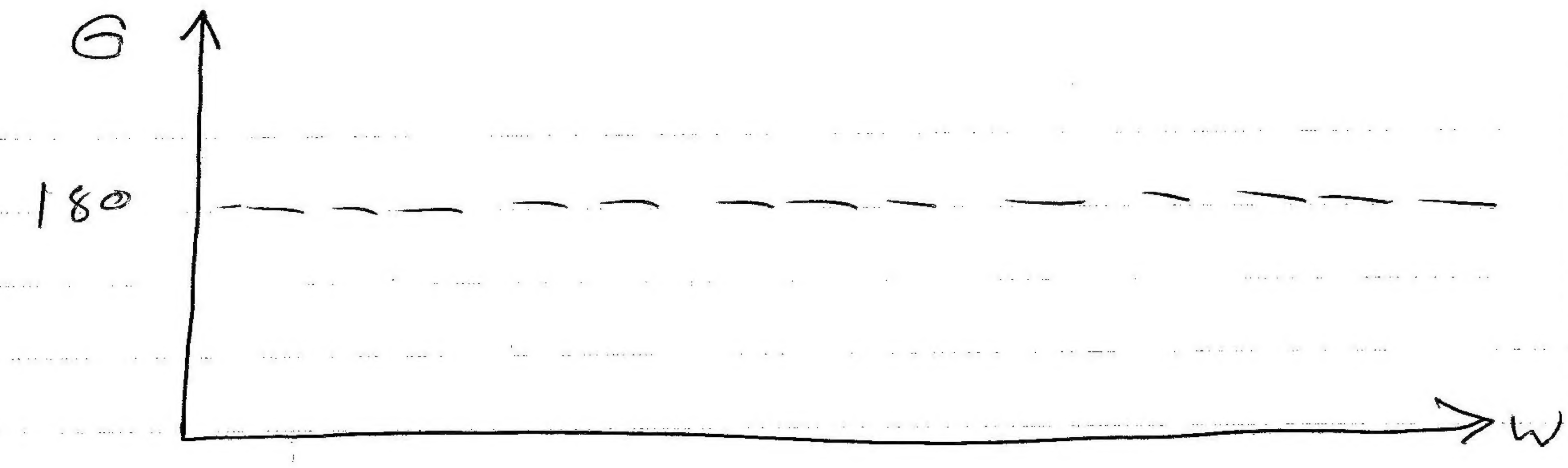
$90^\circ \leftarrow S$

$180^\circ \leftarrow S^2$

$270^\circ \leftarrow S^3$

وهكذا





$\boxed{5.}$ $G(s) = \frac{1}{s}$

$$\boxed{j = \frac{-1}{j}}$$

① $G(s) = \frac{1}{s}$

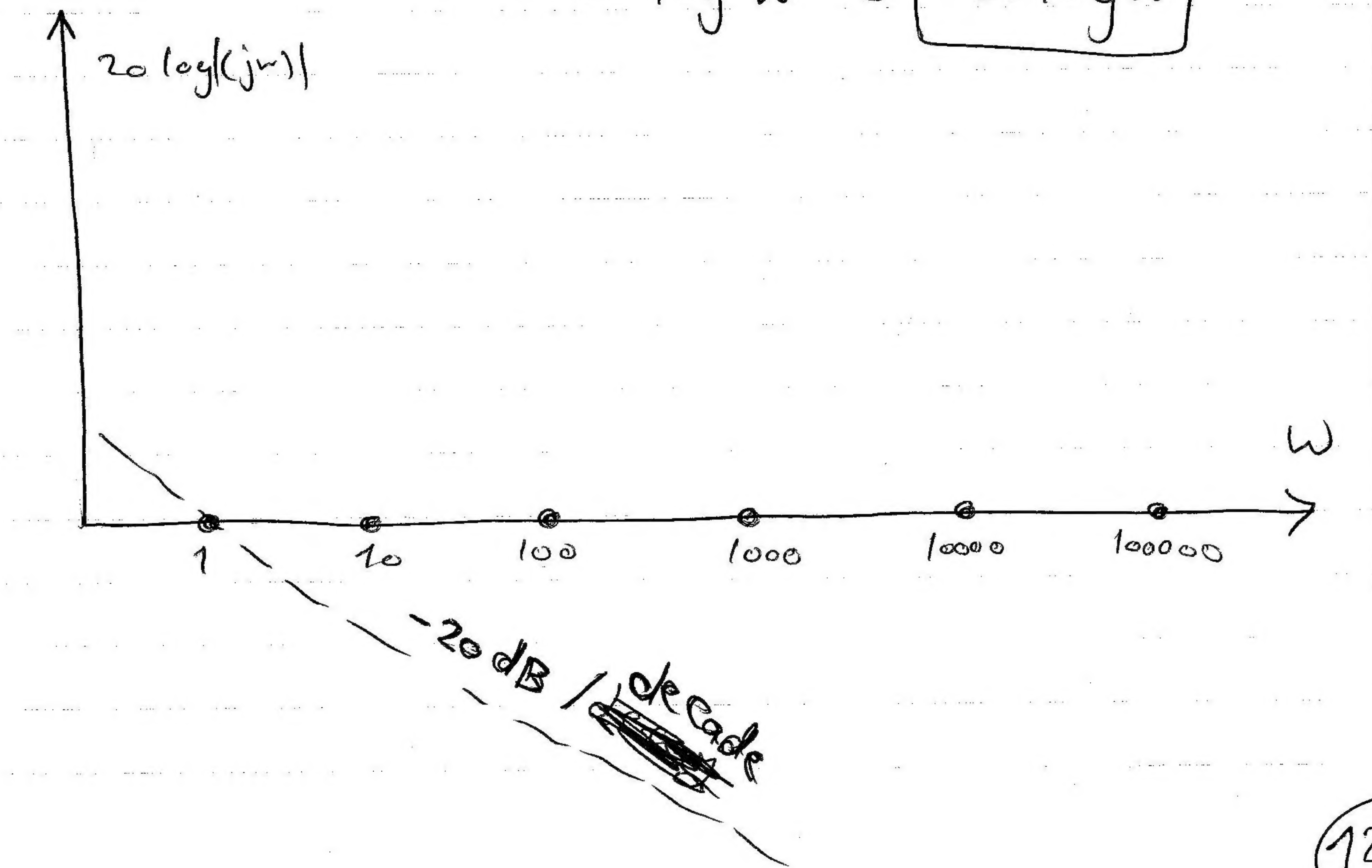
② $G(j\omega) = \frac{1}{j\omega} = \boxed{-\frac{1}{\omega} j}$

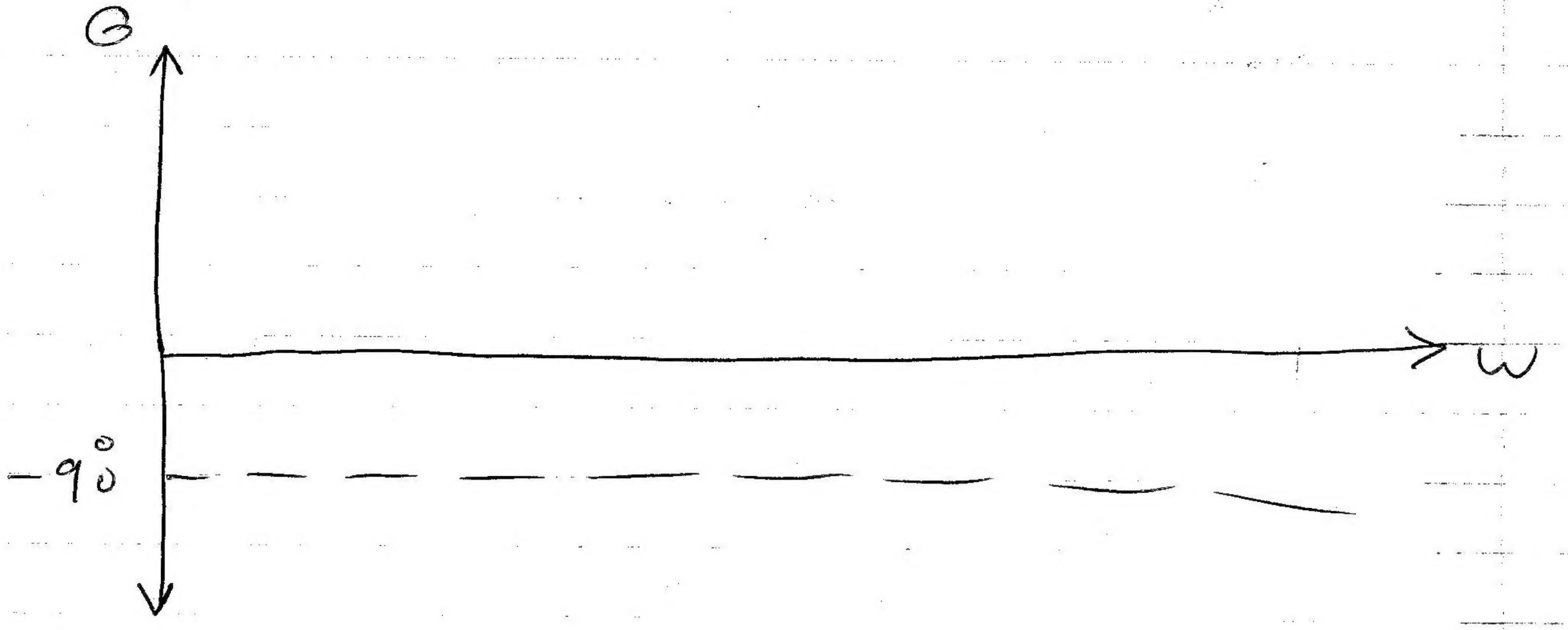
③ $|G(j\omega)| = \frac{1}{\omega}$

$\theta = \tan^{-1} \frac{Im}{Re} = \tan^{-1} \frac{-\frac{1}{\omega}}{0} = \tan^{-1} -\infty = \boxed{-90^\circ}$

④ $20 \log |G(j\omega)| = 20 \log \frac{1}{\omega} =$

$= 20 \log \omega^{-1} = \boxed{-20 \log \omega}$





* كل (S) في البسط تزيد الزاوية بـ phase shift مقداره (90°)

* كل (S) في (المقام) تنقص الزاوية بـ phase shift مقداره (90°)

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6. $G(s) = \frac{1}{s+1}$

① $G(s) = \frac{1}{s+1}$

② $G(j\omega) = \frac{1}{j\omega+1} * \frac{j\omega-1}{j\omega-1} = \boxed{\frac{j\omega-1}{-\omega^2-1}} = \frac{+1}{+(\omega^2+1)} - \frac{j\omega}{\omega^2+1}$

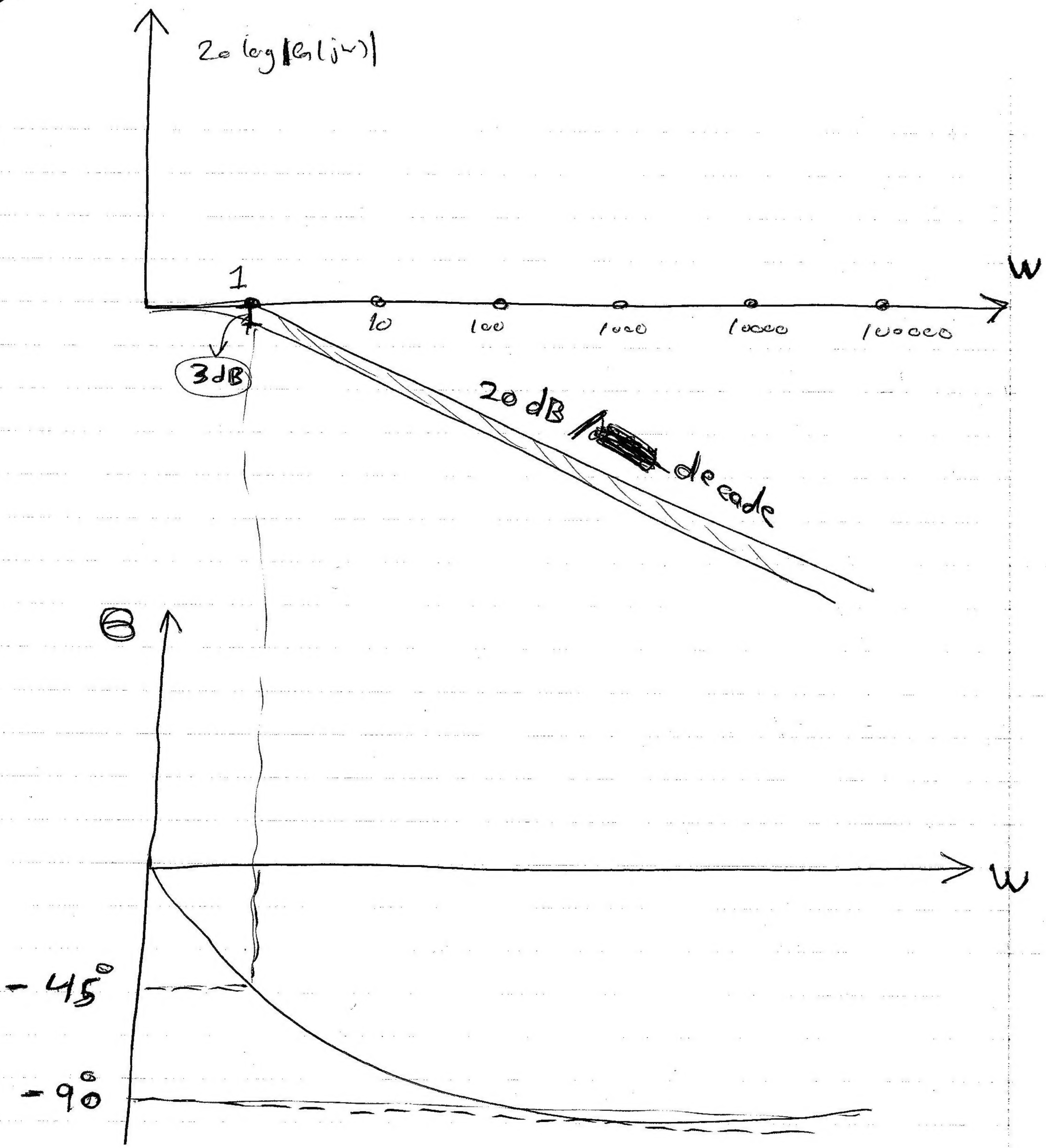
$= \boxed{\frac{1}{\omega^2+1} - \frac{j\omega}{\omega^2+1}}$

③ $|G(j\omega)| \Rightarrow$ ① $\omega \ll 1 \Rightarrow |G(j\omega)| = 1 \Rightarrow \theta = \text{zero}$

② $\omega \gg 1 \Rightarrow |G(j\omega)| = \frac{1}{\omega} \Rightarrow \theta = -90^\circ$

③ $\omega = 1 \Rightarrow |G(j\omega)| = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$

④ $20 \log |G(j\omega)|$



20 log |G(jw)|
w
1
10
100
1000
10000
100000
3dB
20 dB / decade
θ
w
-45°
-90°

Bode magnitude for Complex Systems

$$G(s) = \frac{2S+1}{(S+2)(S+3)(S+4)}$$

$2S+1 \rightarrow w_c = \frac{1}{2}$

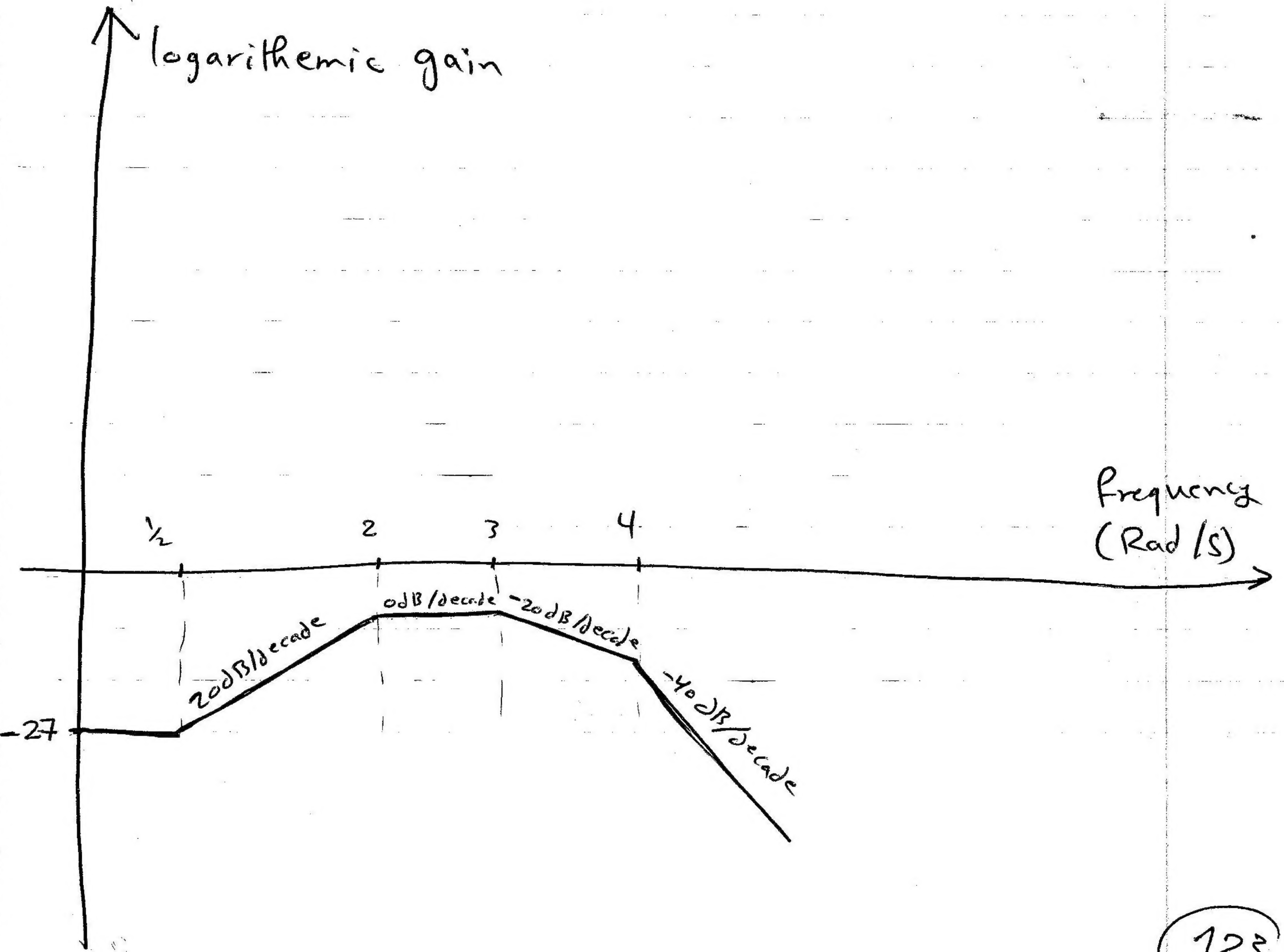
$(S+2) \rightarrow w_c = 2$, $(S+3) \rightarrow w_c = 3$, $(S+4) \rightarrow w_c = 4$

① $\text{Log } G = 20\log 1 - 20\log 2 - 20\log 3 - 20\log 4$

$$= \boxed{20\log\frac{1}{24}} \approx \boxed{-27}$$

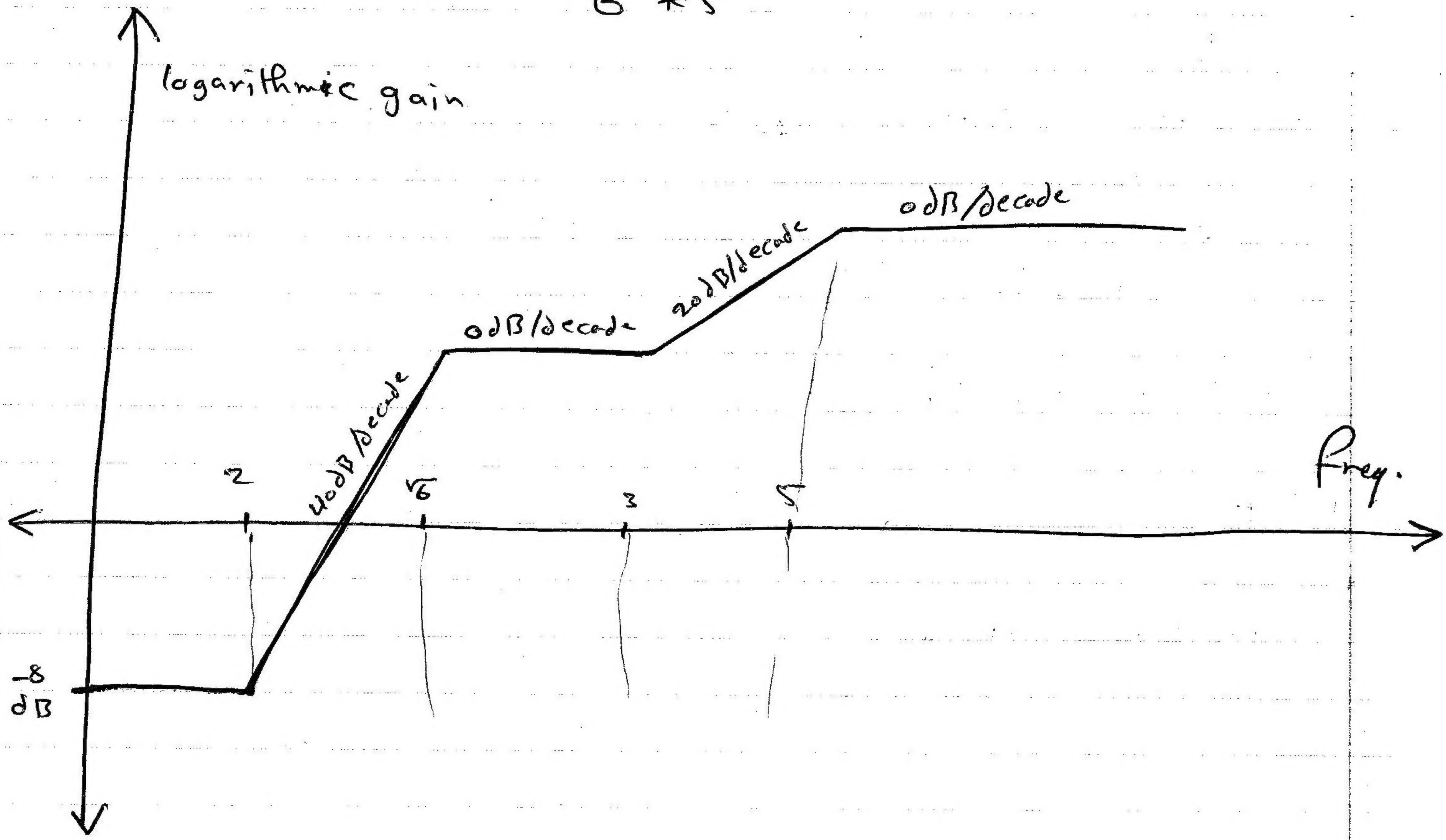
② Find the corner frequencies

③ Plot



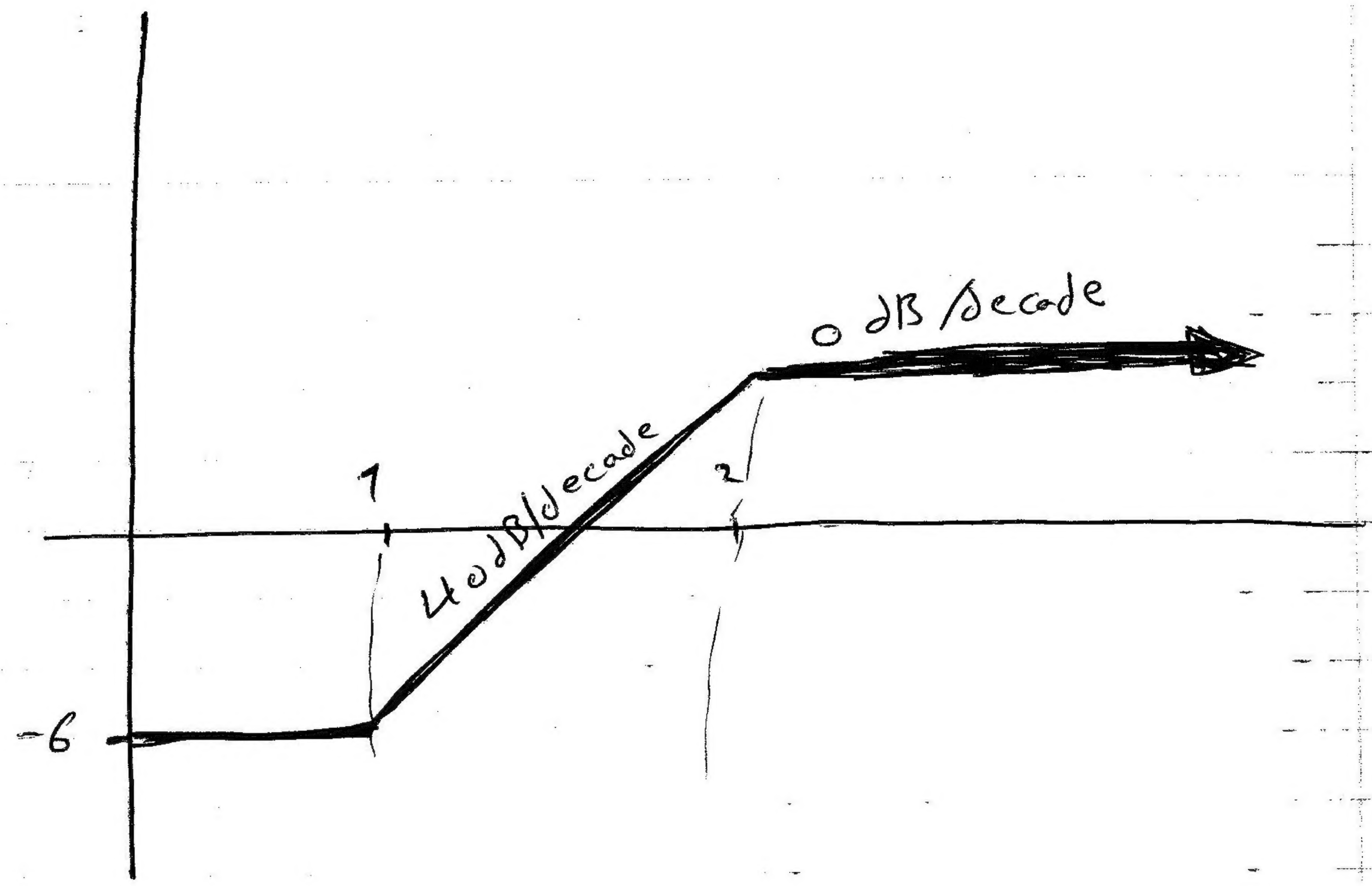
Ex $G(s) = \dfrac{\overset{w_c=2}{(s+2)^2}\,\overset{w_c=3}{(s+3)}}{\underset{w_c=\sqrt{6}}{(s^2+6)}\,\underset{\boxed{w_c=5}}{(s+5)}}$

$\circledast$ $\log o = 20\log\dfrac{2*2*3}{6*5} = 20\log\frac{2}{5} \approx \boxed{-8}$



Ex $G(s) = \dfrac{\overset{w_c=1}{s^2+2s+1}}{\underset{w_c=2}{s^2+2s+4}}$

① $\log o = 20\log\frac{1}{2} = \boxed{-6}$



2. ((Polar Plot))

the graph representing the relationship between Imaginary axis and real axis as frequency is changed.

1. Find $G(jw) = R(jw) + jX(jw)$
2. Find $|G(jw)| \angle \tan^{-1} \frac{X(jw)}{R(jw)}$

   OR make a table of relationship between R & X(jw)
3. plot

Example

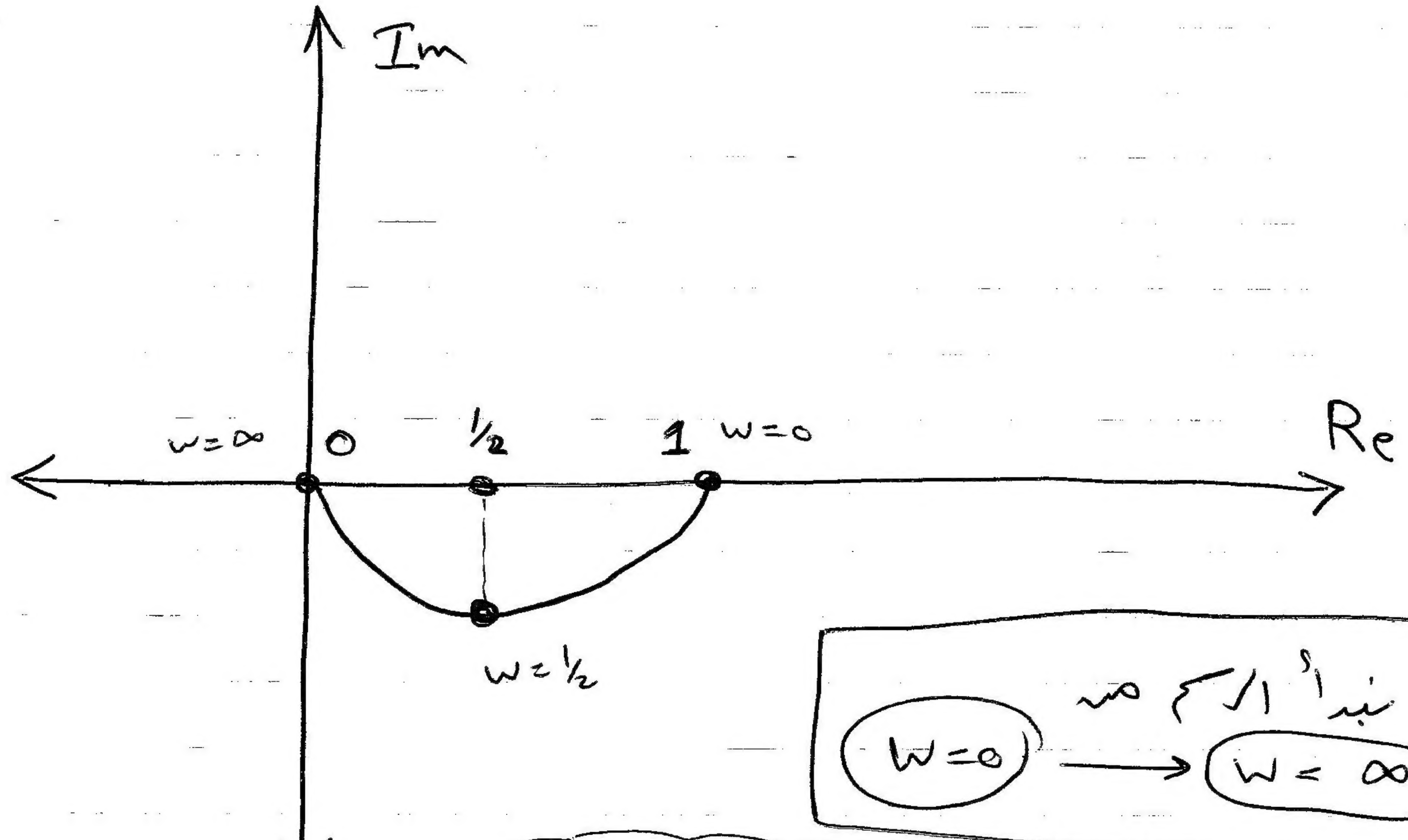
(1.) $G(s) = \dfrac{1}{2s+1}$

(1) $G(jw) = \boxed{\dfrac{1}{2jw+1}} * \left(\dfrac{2jw-1}{2jw-1}\right)$

$= \boxed{\dfrac{2jw-1}{-4w^2-1}} = \underbrace{\dfrac{1}{4w^2-1}}_{\text{Real Part } R(jw)} - \underbrace{\dfrac{2w}{4w^2+1}}_{\text{Imaginary Part } X(jw)} j$

(2)

W	0	1/2	∞
R(jw)	1	1/2	0
X(jw)	0	-1/2	0



نبدأ الرسم من

$W=0 \longrightarrow W=\infty$

$\text{Gain} = \sqrt{\tfrac{1}{2}^2 + \tfrac{1}{2}^2} = \sqrt{\tfrac{1}{4}+\tfrac{1}{4}} = \sqrt{\tfrac{1}{2}} = \boxed{0.7}$

$\tfrac{1}{2} P = 0.7\,V = -3\,dB$